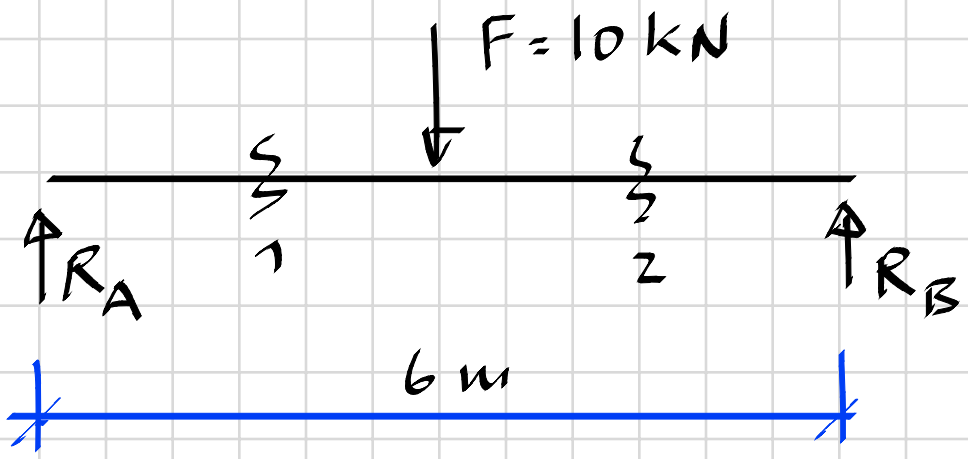
1. **Fritt upplagd balk med punktlast i mitten**
Balklängd: 6 m
Stöd: Enkla stöd i båda ändar
Last: 10 kN punktlast i mitten

1.



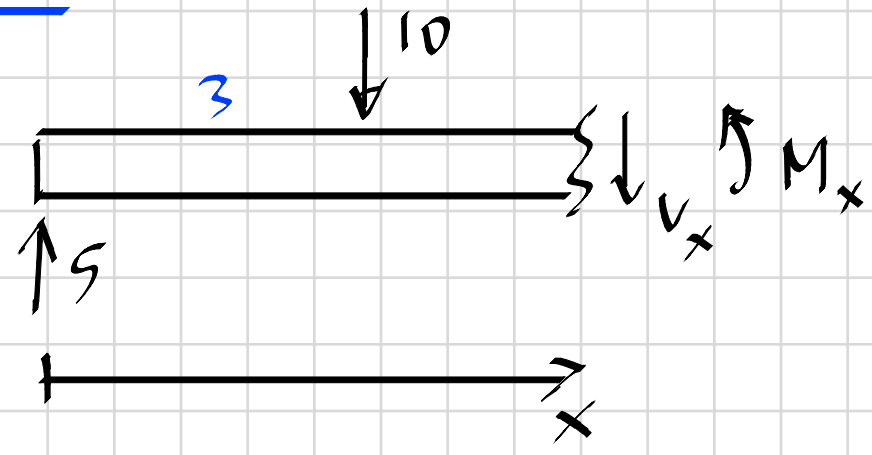
$\uparrow$: $R_A = R_B = 5\ kN$

Snitt 1:

$\uparrow$: $5 - V_x = 0 \Rightarrow V_x = 5$

$\circlearrowleft$: $M_x - 5 \cdot x = 0 \Rightarrow M_x = 5x$

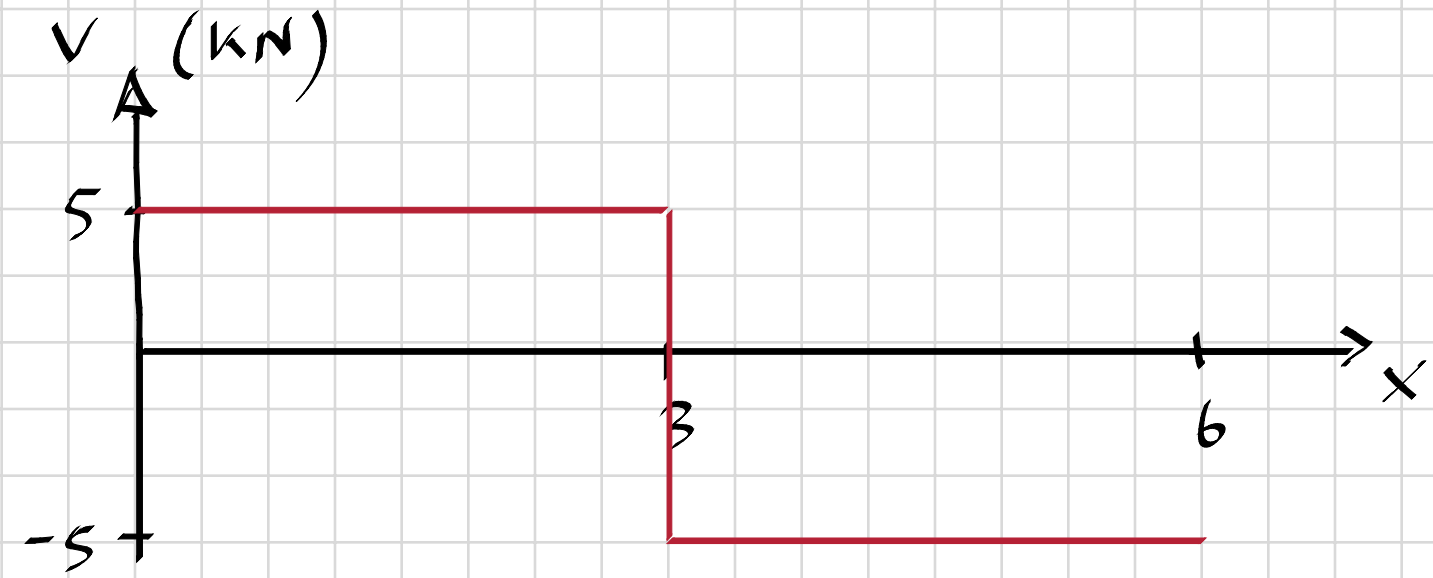
Snitt 2:

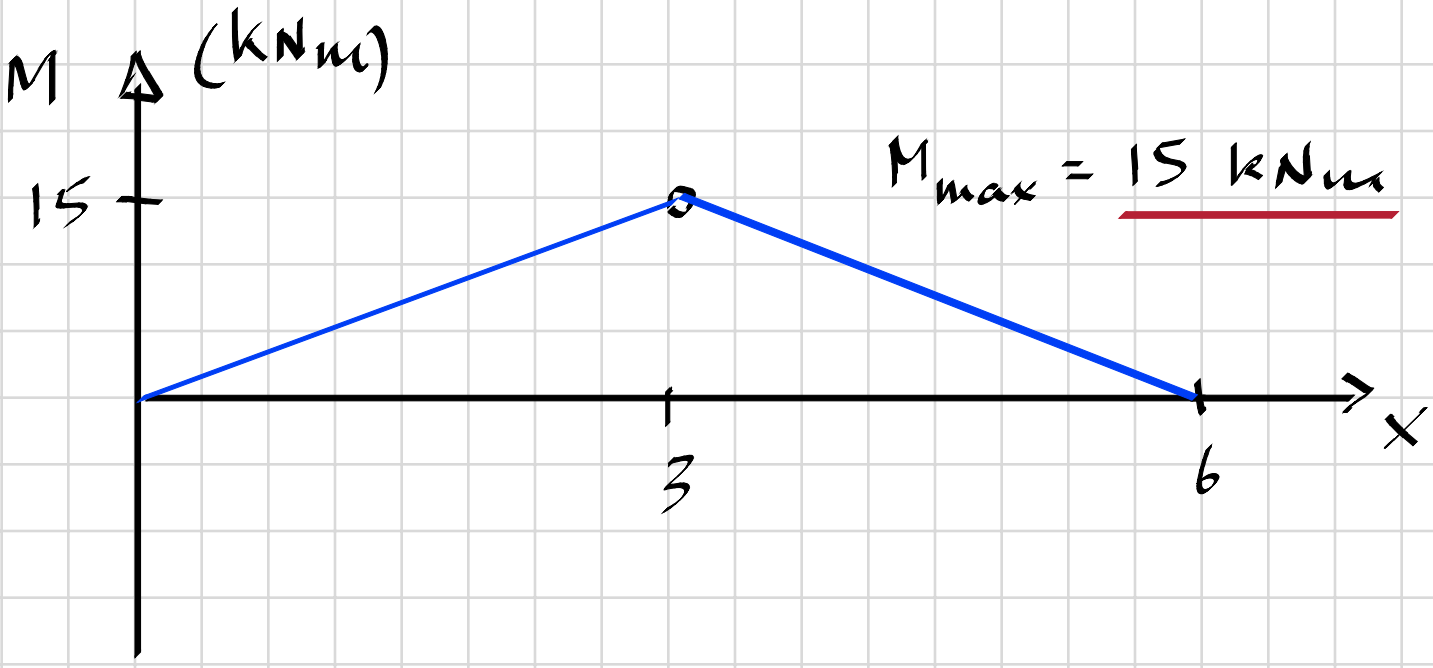


$\uparrow$: $5 - 10 - V_x = 0 \Rightarrow V_x = -5$

$\circlearrowleft$: $M_x - 5x + 10(x-3) = 0 \Rightarrow M_x = -5x + 30$

Kraft- och momentdiagram



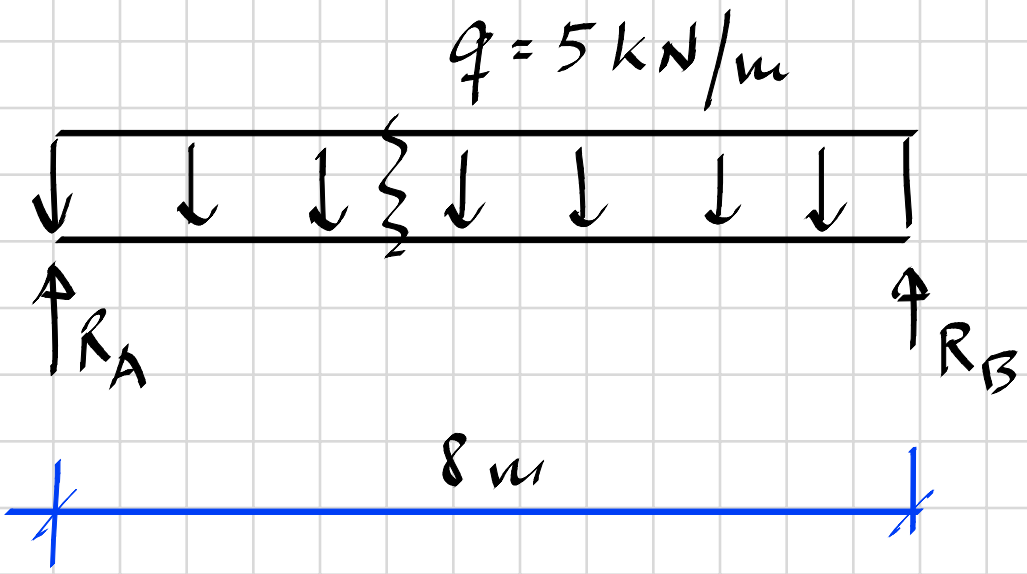


Elementarfall 5 (KB sid 29)

$$M_{max} = \frac{FL}{4} = \frac{10 \cdot 6}{4} = 15 \text{ kNm}$$

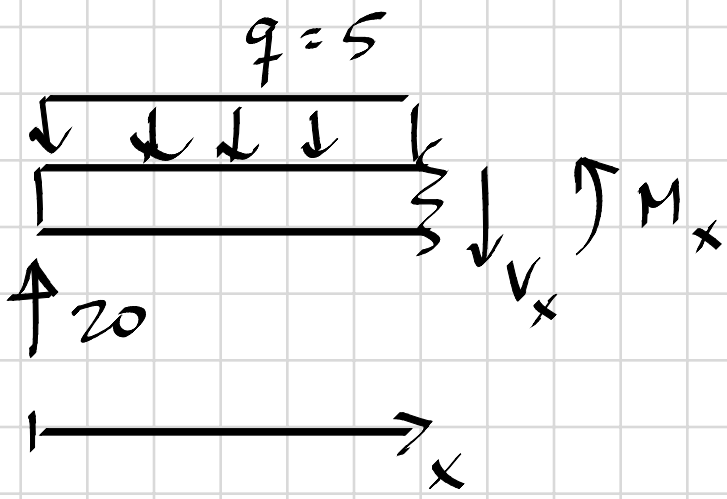
2. **Fritt upplagd balk med jämnt fördelad last**
Balklängd: 8 m
Stöd: Enkla stöd i båda ändar
Last: 5 kN/m jämnt fördelad över hela balken

2.



$\uparrow: \quad R_A = R_B = \frac{5 \cdot 8}{2} = 20 \text{ kN}$

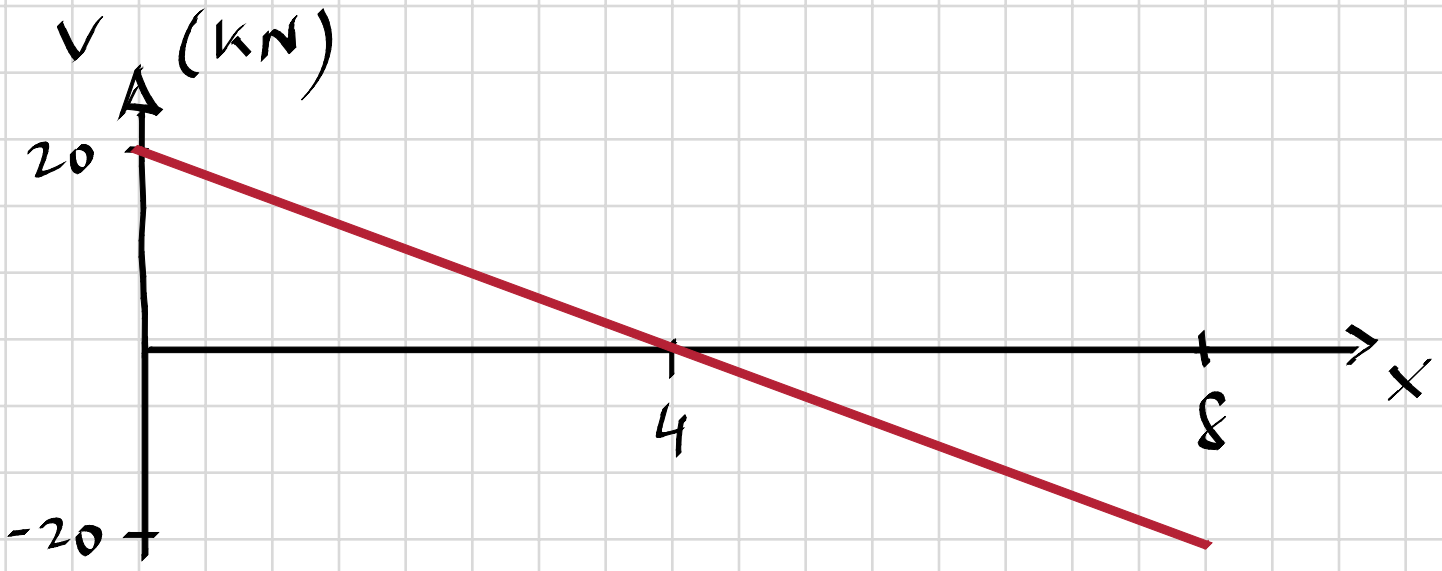
Snitt:

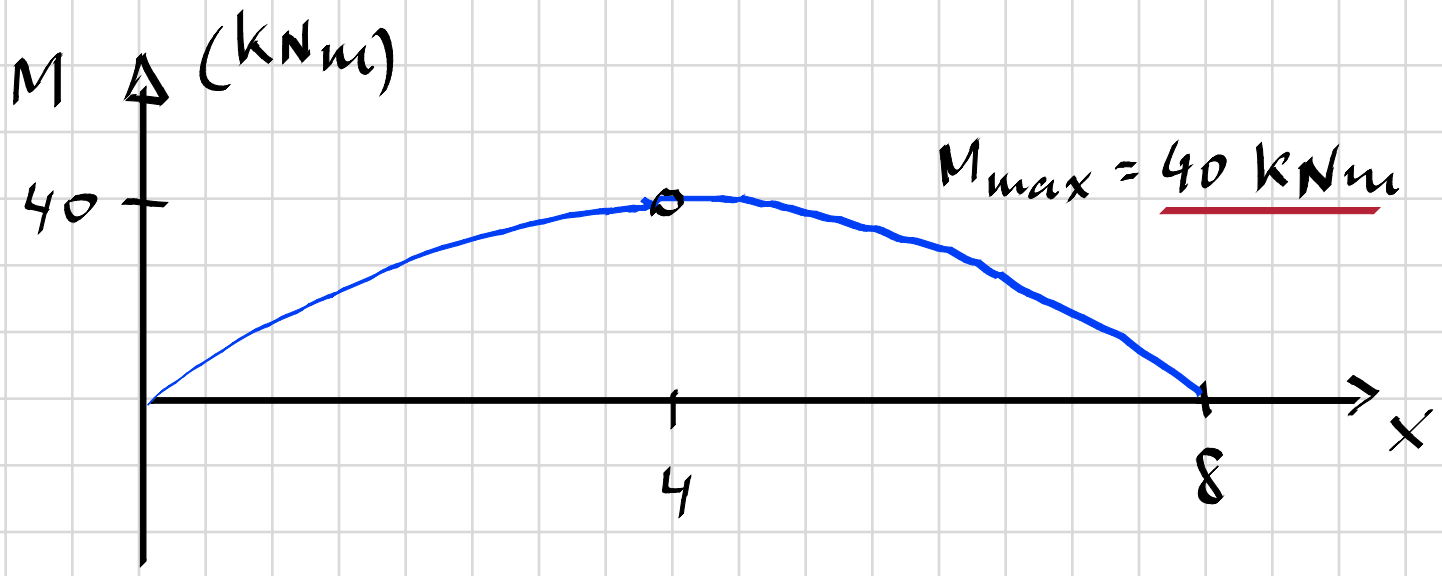


$\uparrow: \quad 20 - 5x - V_x = 0 \quad \Rightarrow V_x = 20 - 5x$

$\circlearrowright: \quad M_x + 5x \cdot \frac{x}{2} - 20x = 0 \quad \Rightarrow M_x = 20x - 2.5x^2$

Kraft- och momentdiagram



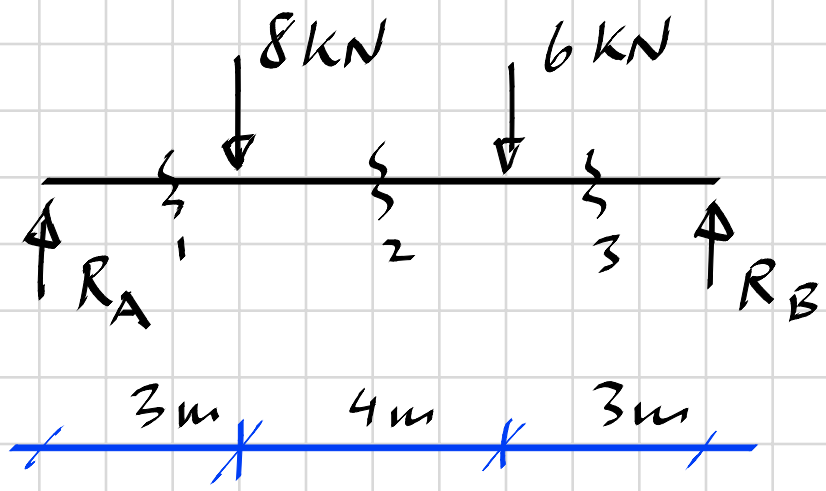


Elementarfall 10 (KB sid 30)

$$M_{max} = \frac{QL}{8} = \frac{q \cdot L^2}{8} = \frac{5 \cdot 8^2}{8} = 40 \text{ kNm}$$

3. **Fritt upplagd balk med två punktlaster**
Balklängd: 10 m
Stöd: Enkla stöd i båda ändar
Last: 8 kN vid 3 m, 6 kN vid 7 m

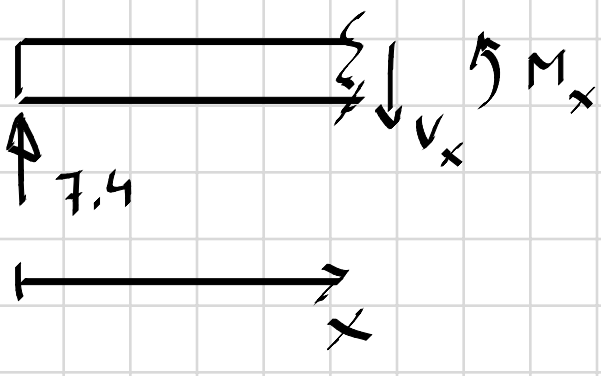
3.



$\circlearrowleft$: $8 \cdot 7 + 6 \cdot 3 - R_A \cdot 10 = 0 \quad \Rightarrow R_A = 7{,}4 \text{ kN}$

$\uparrow$: $R_A + R_B - 8 - 6 = 0 \quad \Rightarrow R_B = 6{,}6 \text{ kN}$

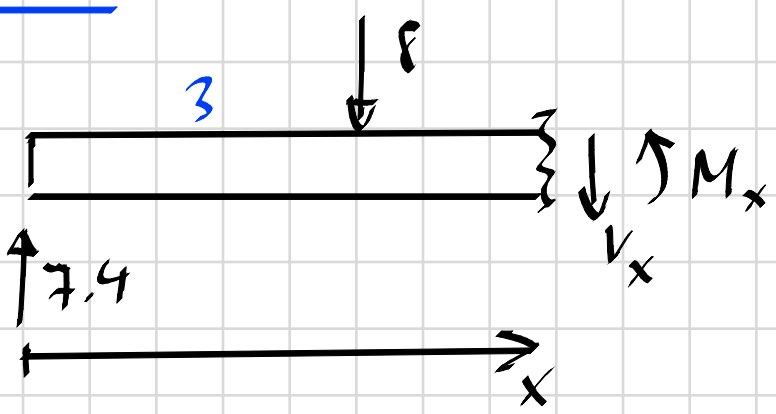
Snitt 1:



$\uparrow$: $7{,}4 - V_x = 0 \quad \Rightarrow V_x = 7{,}4$

$\circlearrowleft$: $M_x - 7{,}4x = 0 \quad \Rightarrow M_x = 7{,}4x$

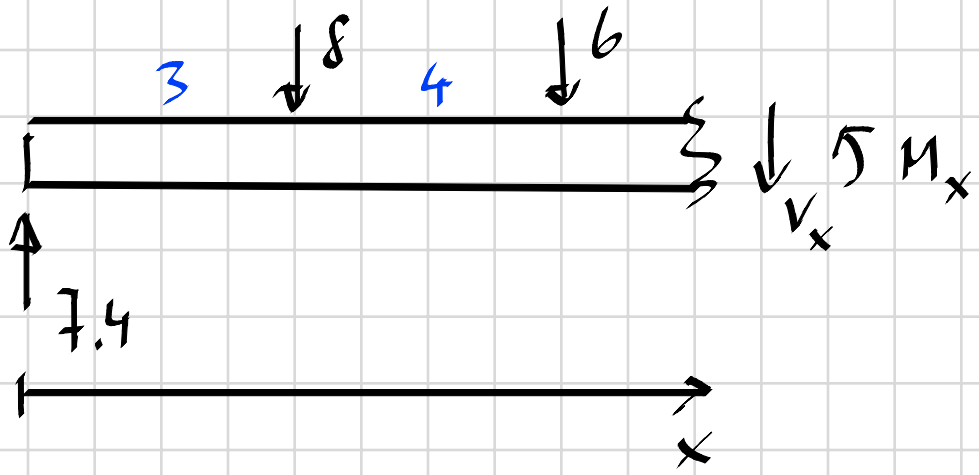
Snitt 2:



$\uparrow$: $7.4 - 8 - V_x = 0 \Rightarrow V_x = -0.6$

$\circlearrowleft$: $M_x + 8(x-3) - 7.4x = 0 \Rightarrow M_x = -0.6x + 24$

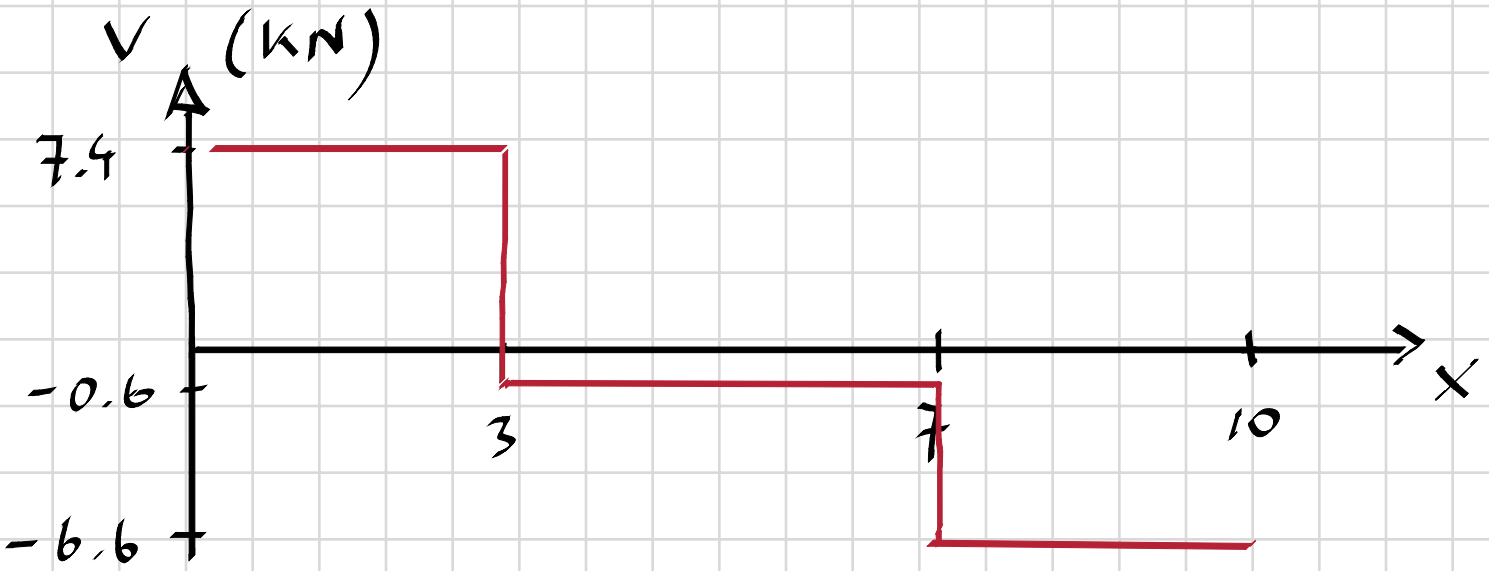
Snitt 3:

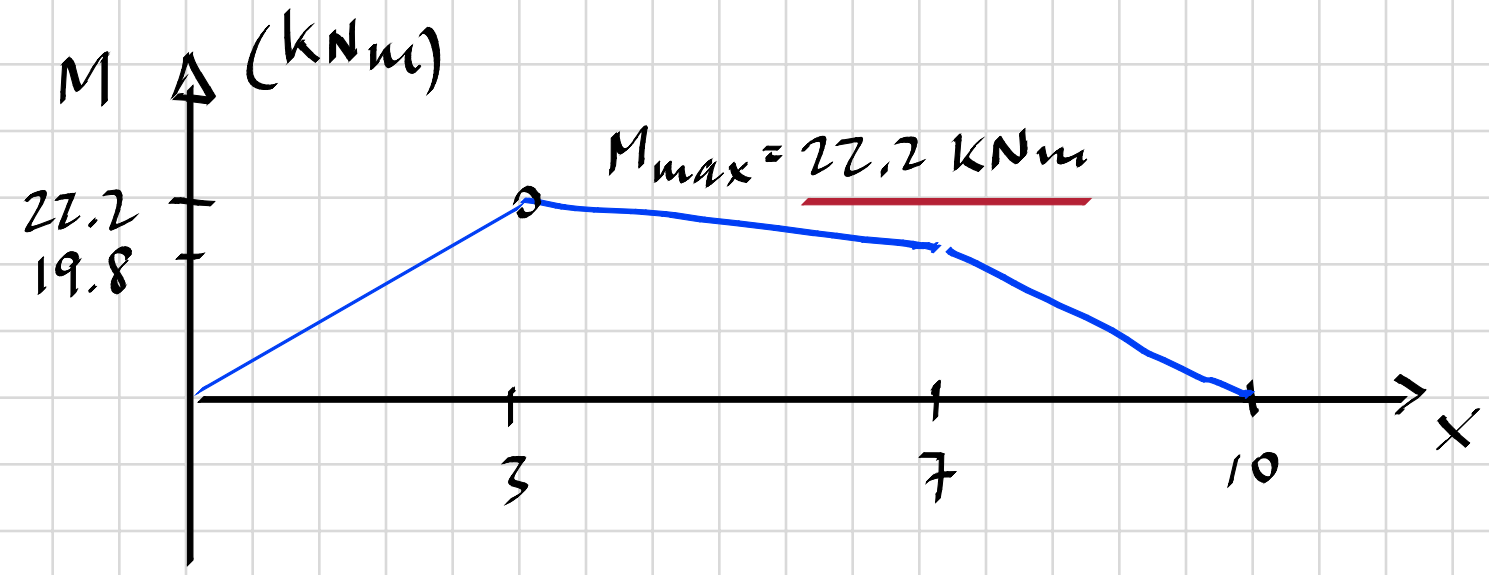


$\uparrow$: $7.4 - 8 - 6 - V_x = 0 \Rightarrow V_x = -6.6$

$\circlearrowleft$: $M_x + 8(x-3) + 6(x-7) - 7.4x = 0 \Rightarrow M_x = -6.6x + 66$

Kraft- och momentdiagram





Elementarfall 6 (KB sid 29)

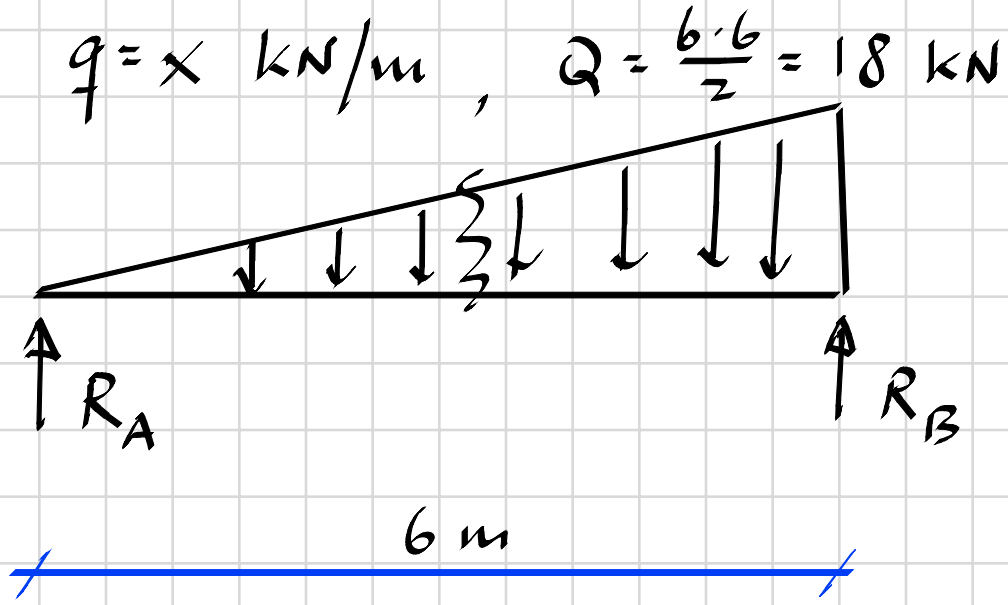
$$M_1 = F_1 \cdot \frac{b_1}{L} \cdot x_{a1} = 8 \cdot \frac{7}{10} \cdot 3 = 16.8 \text{ kNm}$$

$$M_2 = F_2 \cdot \frac{b_2}{L} \cdot x_{a2} = 6 \cdot \frac{3}{10} \cdot 3 = 5,4 \text{ kNm}$$

$$M_{max} = M_1 + M_2 = 16,8 + 5,4 = \underline{22,2 \text{ kNm}}$$

4. **Fritt upplagd balk med triangulär last**
Balklängd: 6 m
Stöd: Enkla stöd i båda ändar
Last: Triangulär last från 0 kN/m till 6 kN/m
över hela balken

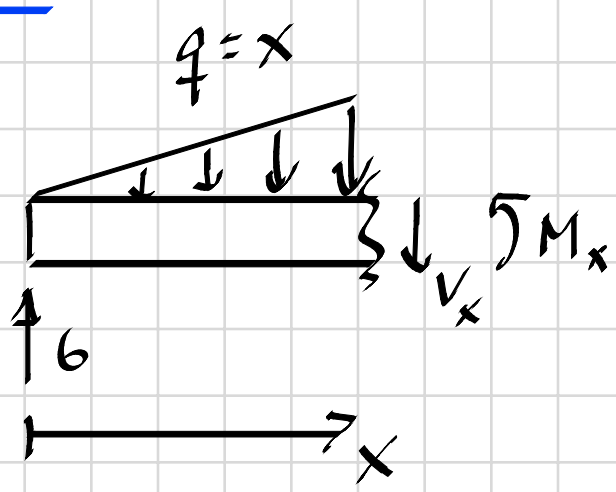
4.



$\circlearrowleft_B$: $18 \cdot 2 - R_A \cdot 6 = 0 \quad \Rightarrow \quad R_A = 6 \text{ kN}$

$\uparrow$: $R_A + R_B - 18 = 0 \quad \Rightarrow \quad R_B = 12 \text{ kN}$

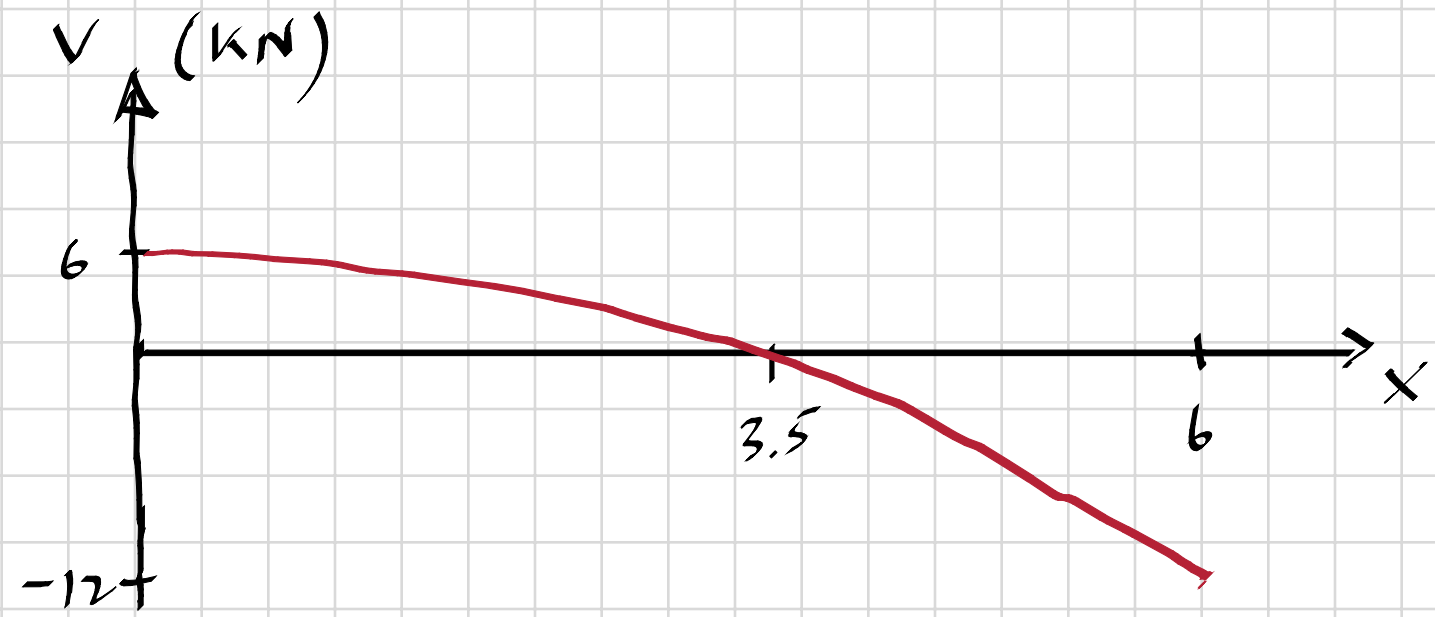
Snitt:

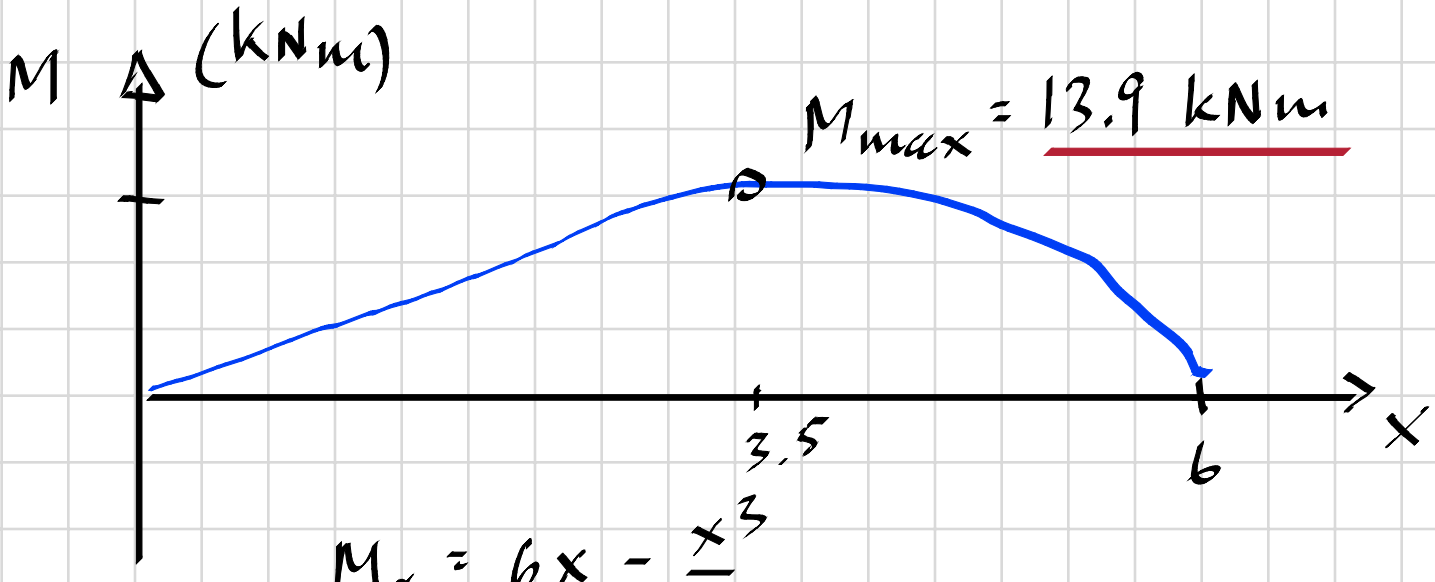


$\uparrow$: $6 - q \cdot \frac{x}{2} - V_x = 0 \quad \Rightarrow \quad V_x = 6 - \frac{x^2}{2}$

$\circlearrowleft$: $M_x - 6x + q \cdot \frac{x}{2} \cdot \frac{x}{3} = 0 \quad \Rightarrow \quad M_x = 6x - \frac{x^3}{6}$

Kraft- och momentdiagram





$$M_x = 6x - \frac{x^3}{6}$$

$$M'_x = 6 - \frac{x^2}{2}$$

$$M'_x = 0 \Rightarrow x = \pm\sqrt{12} \approx 3.5 \text{ m}$$

Elementarfall 11 (KB sid 30)

$$M_{max} = \frac{2QL}{9\sqrt{3}} = \frac{2 \cdot 18 \cdot 6}{9 \cdot \sqrt{3}} = 13.9 \text{ kNm}$$